# DEVELOPMENT OF A NEW METHOD IN FORCE LIMITED VIBRATION TESTING

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The force limited vibration test approaches discussed in NASA-HDBK-7004C were developed to reduce overtesting associated with base shake vibration tests of aerospace hardware. This handbook outlines several different methods of specifying force limiting specifications. The basis for deriving force limiting specifications is related to the differences between mechanical impedances of typical aerospace hardware interfacing the mounting structures in flight configurations and the large impedances inherent in shaker vibration testing. Among these approaches, the semi-empirical method is presently the most widely used method to derive the force limiting specifications. This method is developed based on the assumption that the hardware interface responses are coherently excited. The inclusion of the incoherent excitation of the aerospace structures at mounting interfaces provides the basis for developing a more realistic force limiting specification that may be used to qualify flight hardware for launch environments using shaker testing. In this paper the semi-empirical method for defining force limiting specifications discussed in NASA handbook are reviewed using data recently obtained from a series of acoustic and vibration tests. A mathematical formation of a new force limiting approach based on considering the incoherent excitation of the hardware at their mounting interfaces is developed. Results from the analysis, which are correlated to the data obtained from a series of well instrumented acoustic and vibration tests, are discussed. The new approach provides much more realistic force limits that may further remove conservatism inherent in shaker vibration testing not accounted for by methods discussed in the NASA handbook.

Keywords: ACOUSTIC, VIBRATION, SHAKER TEST, FORCE LIMIT

## 1. Introduction

JPL has pioneered the development of the force limited vibration testing method and has promoted it as a preferred practice in vibration qualification testing of flight hardware within NASA Centers and in the aerospace industry[1-2]. The main attribute of this method is that it alleviates the over-test inherent in conventional shaker vibration testing. The over-test is due to the infinite impedance of the shaker interface as compared to the relatively compliant flight configuration installation of the component undergoing vibration testing. In this method, automatic notching of the input acceleration specification is implemented when force transducers are integrated into the test set up and a force limit specification is included in the shaker control system. Currently, almost all flight hardware qualification vibration tests force limiting specifications are defined using semi-empirical methods[1]. Due to the absence of the flight configuration structural information (the support structure apparent masses and cross-correlation of the component interface responses), engineering judgment and experience are used in deriving the force specification. A new approach, using analytical models, is proposed that accurately takes into account the coupled support structure and component dynamics of flight configuration (i.e. coupled frequency, cross-correlation of the interface responses using six degrees of freedom, and impedances). This method provides a significant improvement in defining the force limiting specification. Unlike the semi-empirical method that relies on the users' experience and judgment and only addresses impedance mismatch in the axis of shake, the proposed method is a physics-based approach that predicts the force spectrum based on the flight configuration finite element models and accounts for the components interface cross-correlation responses[3-4].

The differences between the semi-empirical method currently used in force limited random vibration testing and the component interface force responses in flight configuration can be examined using a well-controlled acoustics and random vibrations tests conducted at JPL a few years ago. The differences between the semi-empirical force limiting specification and the actual measured force spectrum at the box interfaces in a flight-like configuration obtained from an acoustic test were shown to be significant. The conventional force limiting methods are used to account for the shaker impedance mismatch only in the shake direction is not adequate and the coupling dynamics of the support structure and the component must be considered. This paper summarizes both analytical and experimental findings that help develop physics based approach to accurately predict the force limiting specification.

## 2. Experimental results

Several acoustic tests with various hardware configurations funded by NASA Engineering Safety Center (NESC) a few years ago were performed to examine impedances of the loaded panels[5]. The data from that effort is used to examine the force limiting spectrum. The test articles investigated in the NESC study consisted of a) a freely suspended aluminium (Al) panel of dimensions 37.5"x41"x0.25" and of mass 38.4 lb (exclusive of cables), and b) two structurally flight-like electronic boxes A (17.4 lb) and B (45 lb). Three types of tests were conducted: 1) Acoustic tests, 2) tap tests using a calibrated hammer, and 3) vibration tests using boxes A and B on a shake table (data from Box A will be discussed in this paper). Force gages were installed at every mounting interface between the boxes and the supporting structures, whether mounted on the panels or on the shake table vibration fixture. In every test, whether the panels were loaded with the boxes or unloaded, accelerometers were mounted near the boxes mounting positions and response accelerometers were mounted on the boxes themselves. Figure 1 shows the Al panel and boxes A and B suspended in reverberant acoustic chamber and an image of the box A mounted on the shaker head expander.

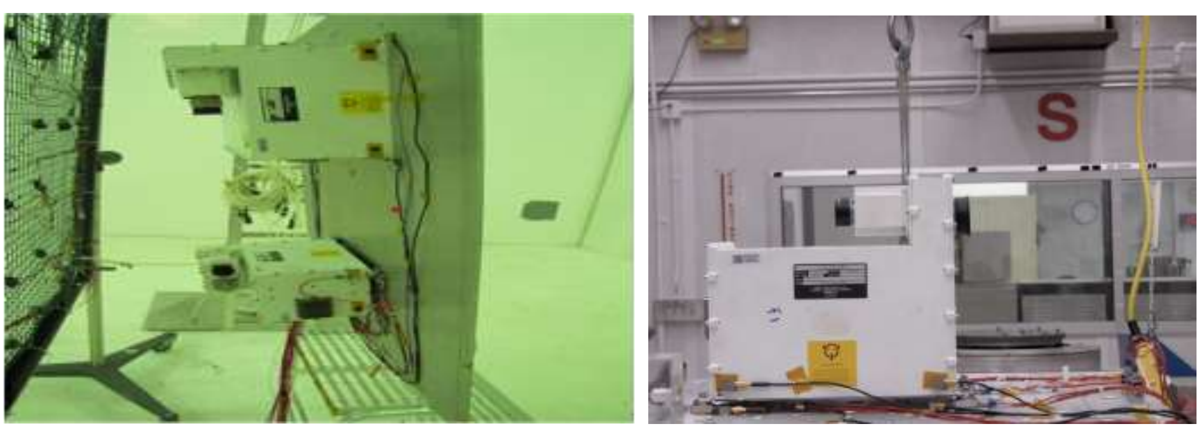

Figure 1: An aluminium panel with two electronic boxes placed inside the JPL acoustic reverberant chamber. Force gages installed at the box/panel interfaces. Electronic box A (right hand photo) mounted on shaker and tested in the vertical direction.

The base shake random vibration tests of the electronic boxes were performed to the average input acceleration spectrum for each box obtained from the acoustic test data. Figure 2 shows the smooth line envelope of the mean acceleration responses of the aluminium panel at box A interfaces. In this figure the mean input acceleration and the vibration test tolerances are shown.

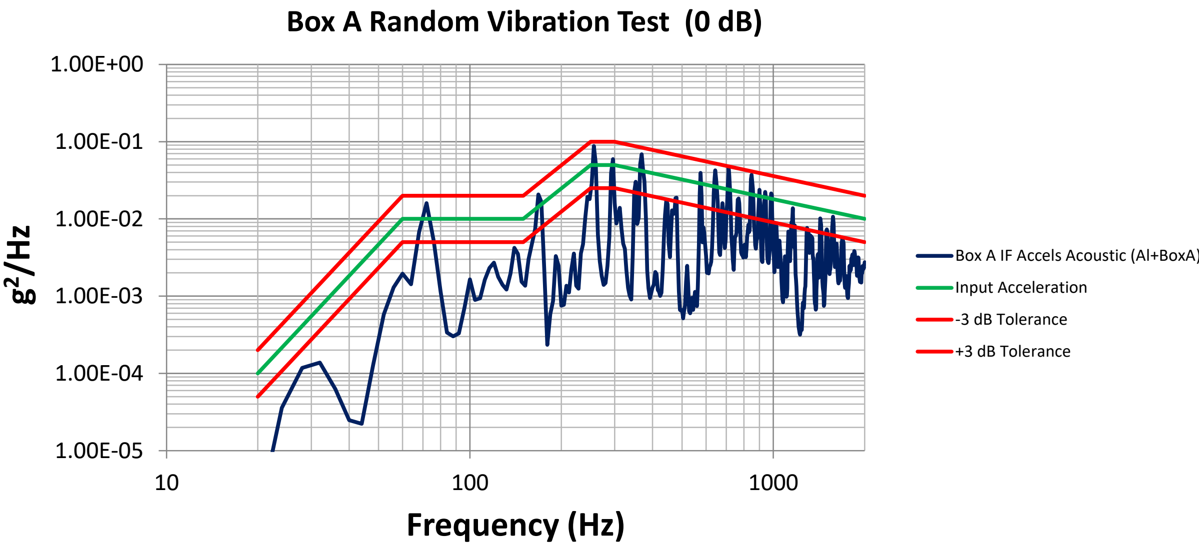


Figure 2: Input acceleration to box A shaker test. The input acceleration spectrum was derived by enveloping the average interfaces acceleration responses obtained from Al panel and Box A acoustic test.

The random vibration environment derived from the acoustic test shown in Figure 2 is used to base shake boxes A and B, separately. Figures 3a and b show the power spectral densities of the average accelerations and summed forces, respectively, measured in the force limited vibration test of box A. The force limit of approximately 100 $lb^2/Hz$ for the vibration test was derived using the semi-empirical equation outlined in the force limiting handbook[1] with an input acceleration spectrum $S_{AA}$ of 0.05 $g^2/Hz$ as shown in Figure 2. Box A had a total weight of 17.9 lbs. The second mode was chosen to be the break frequency, $f_b$, a $C^2$ of 6 was used, and above this frequency the force spectrum slope of 6 dB/Oct was selected to derive the force limiting spectrum. These parameters are typical for force limited vibration testing of most flight hardware. This approach resulted in an approximately 18 dB notch at about 410 Hz, the fundamental resonance of the box A in its test configuration. The maximum force PSD of 10 $lb^2/Hz$ (Figure 3b, green curve) measured in the acoustic test is several orders of magnitude less than the maximum un-notched force response of about 8,000 $lb^2/Hz$ at approximately 410 Hz, the box A frequency, as depicted in Figure 3b. Comparing the force spectral densities of the base shake test and flight-like acoustic test suggests that the uncorrelated interface responses are contributing to the significant differences between the measured force spectra obtained from acoustic tests and the shaker test, where the input for shaker test was obtained from the acoustic test for these cases. The force limited vibration testing performed over the past 20+ years has been based on the differences between the impedances of mounting structures and the large impedances of vibration test shakers and does not account for interface uncorrelated responses of the components.

In the next section mathematical models are developed to predict interface forces in flight configuration that accounts for the uncorrelated responses of the components interfaces.

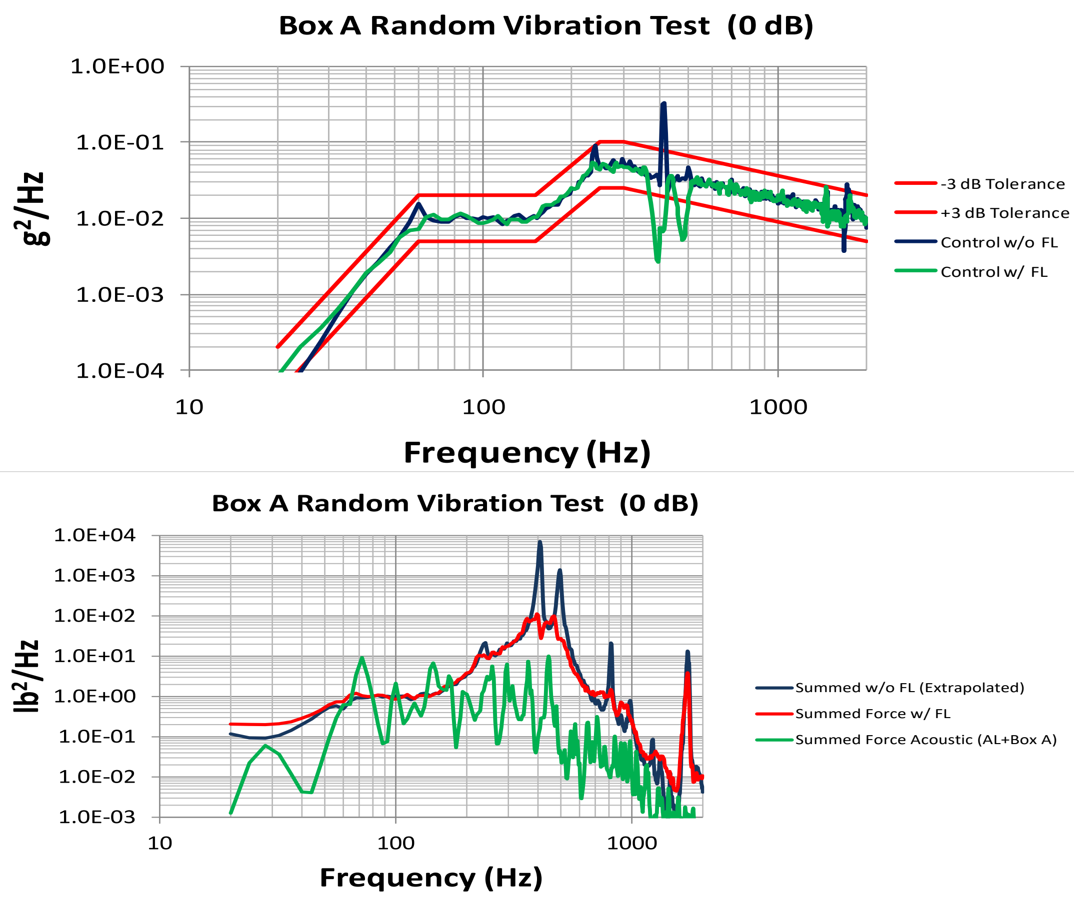


Figure 3: a) Input acceleration to box A shaker test w/ and w/o force limit at 0 dB. The input spectrum w/o force limiting is scaled to full level from -18 dB, and b) the summed box A interface force with and without force limiting at 0 dB (the force spectrum without force limit was scaled to 0 dB from -18 dB), and summed forces measured from acoustic test (green curve). A $C^2$ of 6 was used for this case.

## 3. Force Limiting Mathematical models

The force limiting spectrum of the coupled source and load system is related to the product of the apparent mass and interface acceleration spectrum[6]:

$$FSD_{ii}^{s+l}(\omega) = \mathcal{M}(\omega) ASD_{ii}^{bs}(\omega) \mathcal{M}^{*}(\omega) \tag{1}$$

where $FSD_{ii}^{s+l}$ is the interface force spectral density matrix of the coupled source and load system, $ASD_{ii}^{bs}$ is the interface acceleration spectral density matrix of the unloaded source system, and

$$\mathcal{M}(\omega) = M_{ii}^{l}(\omega)\left[M_{ii}^{s}(\omega) + M_{ii}^{l}(\omega)\right]^{-1} M_{ii}^{s}(\omega),$$

where (2)

$$M_{ii}^{s}(\omega) = \left(A_{ii}^{s}\right)^{-1} \text{ and } M_{ii}^{l}(\omega) = \left(A_{ii}^{l}\right)^{-1},$$

are the "apparent mass" matrices of the source and load system, defined in terms of the inverses of the respective accelerances matrices, $i$ and $j$ partitions are associated with the interface and interior Degrees of Freedom (DOFs), respectively.

The accelerance of the source system is given by

$$\begin{gathered}\ddot{X}^{s}(\omega)=A^{s}(\omega)F^{s}(\omega)\\ \begin{Bmatrix}\ddot{X}_{i}^{s}(\omega)\\ \ddot{X}_{j}^{s}(\omega)\end{Bmatrix}=\begin{bmatrix}A_{ii}^{s}(\omega) & A_{ij}^{s}(\omega)\\ A_{ji}^{s}(\omega) & A_{jj}^{s}(\omega)\end{bmatrix}\begin{Bmatrix}F_{i}^{s}(\omega)\\ F_{j}^{s}(\omega)\end{Bmatrix}\end{gathered} \tag{3}$$

where $F_i^s(\omega)$ and $F_j^s(\omega)$ are the forces and moments applied at the interface and interior DOFs, respectively, and $\ddot{X}_i^s(\omega)$ and $\ddot{X}_j^s(\omega)$ are the corresponding accelerations. Interface DOFs here refers to the interface DOFs to the load system. Also, note that $F_j^s(\omega)$ represents the forces due to an external source, such as an acoustic excitation.

The accelerance of the load system is given by

$$\begin{gathered}\ddot{X}^{l}(\omega)=A^{l}(\omega)F^{l}(\omega)\\ \begin{Bmatrix}\ddot{X}_{i}^{l}(\omega)\\ \ddot{X}_{j}^{l}(\omega)\end{Bmatrix}=\begin{bmatrix}A_{ii}^{l}(\omega) & A_{ij}^{l}(\omega)\\ A_{ji}^{l}(\omega) & A_{jj}^{l}(\omega)\end{bmatrix}\begin{Bmatrix}F_{i}^{l}(\omega)\\ 0\end{Bmatrix}\end{gathered} \tag{4}$$

For the load system, forces and moments at the interior DOFs are assumed to be zero; i.e., $F_j^l(\omega)=0$. The only non-zero forces and moments occur at the interface to the source system; i.e., $F_i^l(\omega)\neq 0$.

## 4. Predicted force limiting specifications

The accelerance matrixes for the source (i.e. panels), load (boxes A and B), and coupled systems (panel and boxes), respectively are inverted to estimate the apparent masses. Figure 4a and b show the predicted apparent masses of the box A and coupled box A and Al panel with their corresponding phases, respectively. All terms in the the inverted accelerance matrixes are summed to obtain the apparent masses shown in these figures. The plateau apparent mass curve is predicted to be close to 17.4 lbf, 37.7 lbf, and 82.1 lbs, for box A and loaded panel.

The accelerances were computed by analytically exciting the panels at the boxes attachment interfaces and computing the resulting vibration responses at the boxes interfaces. In the case of Al panel loaded with boxes A and B, all four interface points for each box were constrained using RB2 elements in the FE model. The base shake analysis of the bare Al panel, panel with box A, panel with box B, and panel with boxes A and B were performed by applying constant acceleration spectral density at each interface. The input acceleration PSD with a constant energy (white noise) was applied at each of the boxes interfaces and the resulting acceleration and force responses were computed at the boxes interfaces.

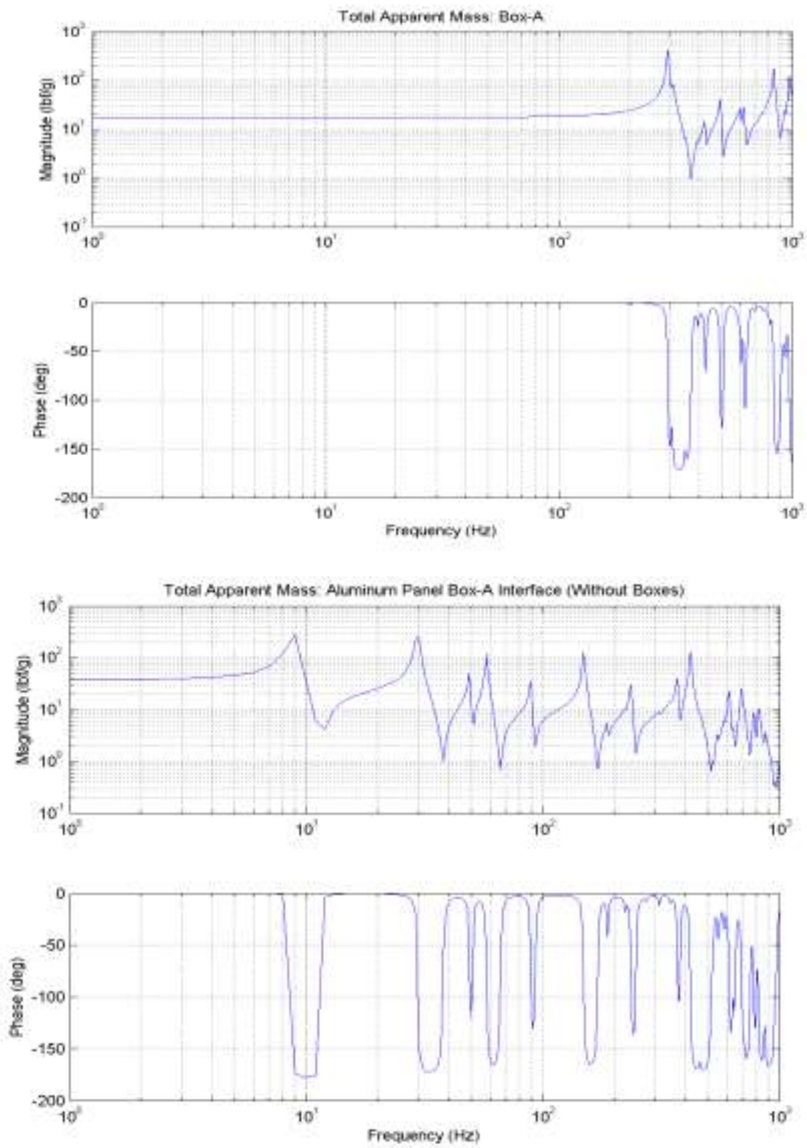


**Figure 4:** Predicted apparent mass of the load (Box A) is computed by inverting the complex accelerance matrix. Predicted apparent mass of the source (Al panel) at box A interfaces is computed by inverting the complex accelerance matrix. All terms in the matrix are used to obtain the apparent mass shown in this Figure.

The first method of predicting force limiting spectrum is to use Eq. (1), where FE models are used to obtain the apparent masses discussed above and shown in Figure 4 of the source and load in flight configurations. Figure 5 show the predicted forces of the loaded Al panel at box A. The semi empirical force limiting is also shown in Figure 5, which clearly shows about 10 dB differences in the force spectrum obtained using conventional approaches. Modal damping of 1% is used in the FEM analysis for all modes. Knowledge of accurate damping will result in more realistic predicted force limiting specifications.

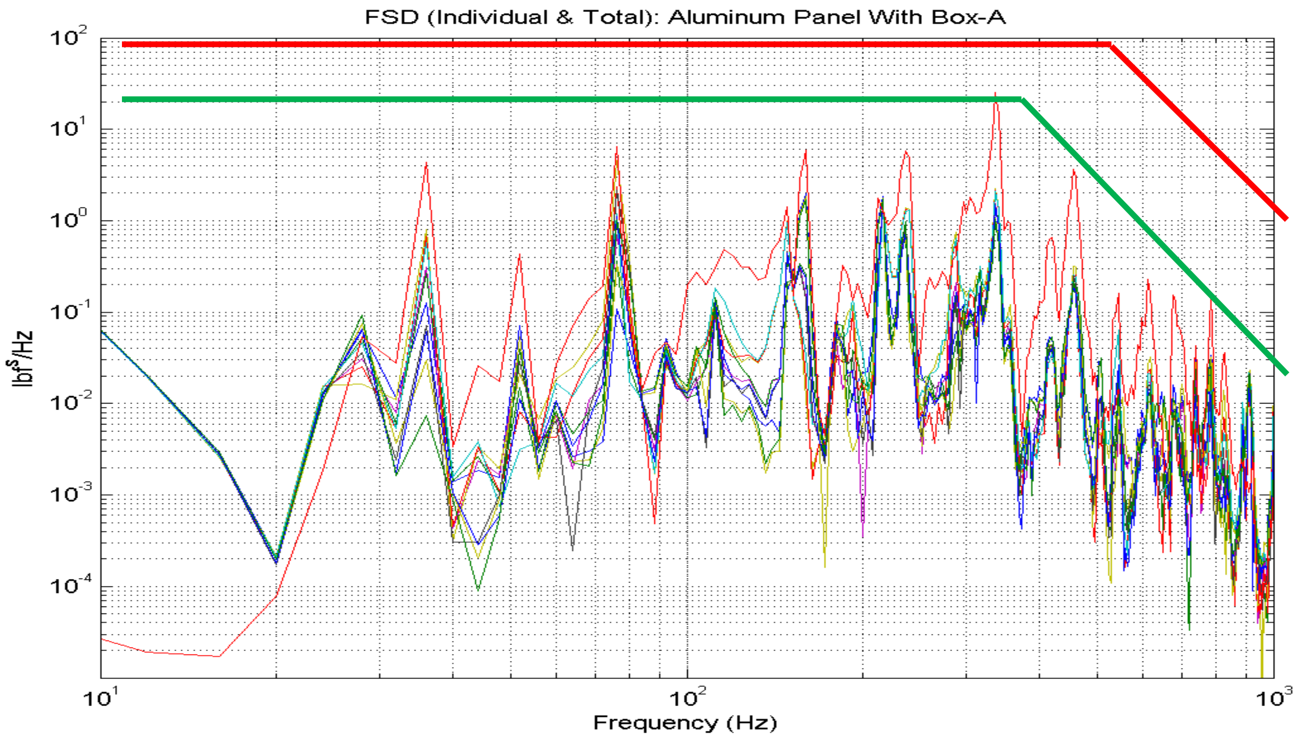


Figure 5: The FEM predicted forces using Eq. (1) and apparent masses shown in Figure 4 at box A interfaces mounted on Al panel. The envelop of the predicted forces (green line) are compared with the semi-empirical derived force limiting specification (red line).

The acoustic induced vibration in flight configuration may also be obtained by applying boundary element method (BEM) analysis to predict the component interfaces forces excited by the acoustic field. The envelop of the BEM predicted forces provides the acoustic induced vibration and a more accurate force limiting specification than the one used in vibration testing of flight hardware.

## 5. Summary: Proposed revision to NASA Handbook 7004C.

A new method in defining force limiting specification based on the uncorrelated dynamics responses of equipment mounted on panel are developed. The new method is based on predicting the impedances of the source and load and estimating the force responses in flight configuration using Eq. (1). It is proposed to amend section 5.8 "FEM Analysis of Force Limits" of NASA-HDBK-7004C[1] "Force Limited Vibration Testing" to incorporate results from this study summarized above. The following NASA recommended practices for FEM force limiting specification are proposed to be added:

1. Use finite element models of the source and components in flight configuration to predict loaded interfaces using the auto and cross-correlation with all DOFs. The predicted forces are obtained either by knowing the source acceleration responses at components' interfaces or assuming unit interface forces (i.e. white noise)
    a. Envelop the force summed over all elements in the apparent mass matrix
        i. Enveloped force specification removes the potential inaccuracy in predicting the coupled system's modes,
    b. Add 3-dB to the enveloped spectrum to account for uncertainties in modeling and additional margin as needed to account for future configuration changes,
    c. Update the predicted force specifications as the FEMs are matured.

2. Use Boundary Element Method analysis to predict the interface forces of the component mounted on source structures
    a. Envelop the force summed over all interface forces keeping real and imaginary,
    b. Add 3-dB to the enveloped spectrum to account for uncertainties in modeling and additional margin as needed to account for future configuration changes,
    c. Update the predicted force specifications as the FEMs are matured.

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